Decidability of Bisimulation for BPA with deadlock and the empty process

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1 Introduction

2 $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ and Context-Free Processes

The class of recursive $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ processes is defined by the following abstract syntax:

$$P ::= \mathbf{0} \mid \mathbf{1} \mid a.P \mid X \mid P + P \mid P \cdot P$$

where a ranges over the finite set of atomic actions Act, and X ranges over the finite set of variables $\mathcal{V}$.

Introduce transparency, guardedness. $\odot$

Definition 2.1 (Guardedness)**.** … $\odot$

Definition 2.2. A $\mathrm{BPA}_{\mathbf{0},\mathbf{1}}$ *process* is defined by a finite guarded recursive specification. Each such a finite guarded recursive specification corresponds to a finite transition system. We use *structural operational semantics* [?], with the rules given below, to give this correspondence.

$$\frac{}{\mathbf{1}\downarrow} \qquad \frac{}{a.x \xrightarrow{a} x}$$

$$\frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'} \qquad \frac{x\downarrow}{x + y\downarrow} \qquad \frac{y\downarrow}{x + y\downarrow}$$

$$\frac{x \xrightarrow{a} x'}{x \cdot y \xrightarrow{a} x' \cdot y} \qquad \frac{x\downarrow \quad y \xrightarrow{a} y'}{x \cdot y \xrightarrow{a} y'} \qquad \frac{x\downarrow \quad y\downarrow}{x \cdot y\downarrow}$$

$$\frac{t_X \xrightarrow{a} x \quad X = t_X}{X \xrightarrow{a} x} \qquad \frac{t_X\downarrow \quad X = t_X}{X\downarrow}$$

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2.1 Bisimulation equivalence and Greibach normal form

Introduce (several?) bisimulation notions. ⊙

Definition 2.3 (Bisimulation). ... ⊙

Definition 2.4 (Bisimulation approximations). ... ⊙

Using the axioms, any guarded recursive specification E can be brought into *Greibach normal form* [**?**] such that bisimilarity is preserved:

$$X = \sum_{i \in I_X} a_i.\xi_i \ (+\mathbf{1}) \qquad \text{for all } X \in \mathcal{V}_E.$$

In this form, every right-hand side of every equation consists of a number of summands, indexed by a finite set I_X (the empty sum is $\mathbf{0}$), each of which is $\mathbf{1}$, or of the form $a_i.\xi_i$, where ξ_i is the sequential composition of a number of variables (the empty sequence is $\mathbf{1}$).

We define the set of *context-free processes* as the subset of the $\text{BPA}_{\mathbf{0},\mathbf{1}}$ processes that have bounded branching. *(Introduce notion of bounded branching.)* ⊙ If we look at the GNF of $\text{BPA}_{\mathbf{0},\mathbf{1}}$ processes, we can see that if $X_i \xrightarrow{w}\!\!\!\!\rightarrow X_i'$ for $w \in \mathcal{A}^+$, then X_i' is just a sequence ξ_i of variables. This means that each state of the transition system of a $\text{BPA}_{\mathbf{0},\mathbf{1}}$ process is labelled with such a sequence.

We assume for now *(prove!)* that if a process is context-free, every state with be labelled by a sequential composition of variables of which on the last one may be transparent. ⊙

Conjecture: there is also an restrictive GNF for this? ⊙

3 Decidability of Bisimulation Equivalence

3.1 Self-bisimulations

Leave subsection as is? ⊙

Definition 3.1 (Least precongruence $\underset{R}{\longleftrightarrow}^*$). ... ⊙

Definition 3.2 (Self-bisimulation R). ... ⊙

Lemma 3.1. *If R is a self-bisimulation then $\underset{R}{\longleftrightarrow}^* \subseteq R$.*

Corollary 3.1. *$\alpha \sim \beta$ iff there is a self-bisimulation R such that $\alpha \, R \, \beta$.*

3.2 Decompositions

We define the *norm* of a process P, written as $|P|$, is defined as follows:

$$|P| = \min\{\text{length}(w) \mid P \xrightarrow{w}\!\!\!\!\rightarrow P' s.t.\ w \in \mathcal{A}^* \text{ and } P'\downarrow\}.$$

A process P is *normed* if $|P| < \infty$.

We divide the variable set $\mathcal{V}$ into disjoint subsets $\mathcal{V}_{\text{fin}} = \{X \in \mathcal{V} \mid X \text{ is normed}\}$ and $\mathcal{V}_\infty = \mathcal{V} - \mathcal{V}_{\text{fin}}$. We sometimes also distinguish between transparent finitely normed variables $\mathcal{V}_{\text{fin}}^{+\mathbf{1}}$ and opaque finitely normed variables $\mathcal{V}_{\text{fin}}^{-\mathbf{1}}$.

Rewrite: We can consider the cases $\xi_i \in \mathcal{V}_{\text{fin}}\mathcal{V}_\infty \cup \mathcal{V}_{\text{fin}}^{-\mathbf{1}*} \cup \mathcal{V}_{\text{fin}}^{-\mathbf{1}*}\mathcal{V}_{\text{fin}}^{+\mathbf{1}}$ due the fact that everything after an unnormed variable can be thrown away preserving bisimulation. ⊙

Definition 3.3 (Decomposability)**.** *Might need to be adapted:* ⊙
When $X\alpha \sim Y\beta$ we say that the pair $(X\alpha, Y\beta)$ is decomposable if $X, Y \in \mathcal{V}_{\text{fin}}$ and there is a γ such that

- $\alpha \sim \gamma\beta$ and $X\gamma \sim Y$ if $|X| \leq |Y|$
- $\gamma\alpha \sim \beta$ and $X \sim Y\gamma$ if $|Y| \leq |X|$

Lemma 3.2. *If $\alpha \sim X\gamma\alpha$ and $\beta \sim X\gamma\beta$ then $a \sim \beta$.*

Check/redo: looks good, what was the counter-example for the non-restricted sequences case? ⊙

Only consider $\mathcal{V}^{-\mathbf{1}*}\mathcal{V}$ and $\mathbf{1}$ here? We call $\phi \in \mathcal{V}^*$ a *unifier* for $\alpha, \beta \in \mathcal{V}^*$ if $\alpha \not\sim \beta$ but $\alpha\phi \sim \beta\phi$. ⊙

We define the branching degree of a specification E, $\deg(E)$, as the size of the largest set $\{\alpha \mid X \xrightarrow{a} \alpha, a \in \mathcal{A}\}$ of all variable $X \in \mathcal{V}_E$.

Lemma 3.3. *For any $\alpha, \beta \in \mathcal{V}^*$, if $\alpha \sim_n \beta$ then there are at most $(\deg(E))^{n-1}$ different unifiers up to $\sim$.*

Proof. Induction on n using the previous lemma. For the base case if $\alpha \sim_1 \beta$ then without loss of generality $\alpha \xrightarrow{a}$ but $\beta \not\xrightarrow{a}$. But there cannot be a unifier ϕ giving $\alpha\phi \sim \beta\phi$.

No! Counter-example:

$$X = a.X + \mathbf{1}$$
$$Y = b.\mathbf{1} + \mathbf{1}$$
$$Z = a.Z + b.Z + \mathbf{1}$$

Then taking $\alpha = X$, $\beta = Y$, $\phi = Z$, we have $\alpha\phi \sim \beta\phi$.

Check/redo. □ ⊙

Lemma 3.4. *For any $X, Y \in \mathcal{V}$ any set R of the form ... such that all pairs are distinct, is finite.*

Adapt the form. Check/redo. ⊙

⊙

3.3 Finite representability of $\sim$

Introduce the notion of size. This might need to be adapted? Then introduce the well-founded ordering on all possible sequences. ⊙

Theorem 3.1. *There is a finite relation R on* (insert notation for all possible sequences) *such that* $\sim \,=\, \underset{R}{\longleftrightarrow}^{*}$. $\odot$

Check/redo. $\odot$

Definition 3.4. ... $\odot$

Theorem 3.2. *Bisimulation equivalence is decidable for all guarded context-free processes.*