5
Dimensions

5.1 Power of multinational corporations

Critics of globalization often make the following comparison [14] to prove the excessive power of multinational corporations:

> In Nigeria, a relatively economically strong country, the GDP [gross domestic product] is $99 billion. The net worth of Exxon is $119 billion. "When multinationals have a net worth higher than the GDP of the country in which they operate, what kind of power relationship are we talking about?" asks Laura Morosini.

Before continuing, explore the following question:

What is the most egregious fault in the comparison between Exxon and Nigeria?

The field is competitive, but one fault stands out. It becomes evident after unpacking the meaning of GDP. A GDP of $99 billion is shorthand for a monetary flow of $99 billion per year. A year, which is the time for the earth to travel around the sun, is an astronomical phenomenon that has been arbitrarily chosen for measuring a social phenomenon—namely, monetary flow.

Comments on page 1

And this is why we never neglect units...

Excuse me – dimensions.

Err, what? I don't quite follow. Are you saying economic output vs. value are not comparable?

Alright, now it makes more sense after continuing to read - the Arrested Development reference sans context was weird.

I've seen this fallacious comparison before, but I can't remember where...

These comparisons are made all the time about the US nat'l debt.

This is a very bad comparison, but I still feel like thinking that the entire GDP of Nigeria is about enough to buy one Exxon is interesting. Even though it's twisted, and probably not actually useful information.

I don't think people can really picture what the economic output of foreign countries is. Using a state in the US might be a more useful comparison, but that would defeat the purpose of comparing to a foreign country. The main problem is that we can't comprehend the size of Nigeria and what goes on there.

Interesting way to think about it

Suppose instead that economists had chosen the decade as the unit of time for measuring GDP. Then Nigeria's GDP (assuming the flow remains steady from year to year) would be roughly \$1 trillion per decade and be reported as \$1 trillion. Now Nigeria towers over Exxon, whose puny assets are a mere one-tenth of Nigeria's GDP. To deduce the opposite conclusion, suppose the week were the unit of time for measuring GDP. Nigeria's GDP becomes \$2 billion per week, reported as \$2 billion. Now puny Nigeria stands helpless before the mighty Exxon, 50-fold larger than Nigeria.

A valid economic argument cannot reach a conclusion that depends on the astronomical phenomenon chosen to measure time. The mistake lies in comparing incomparable quantities. Net worth is an amount: It has dimensions of money and is typically measured in units of dollars. GDP, however, is a flow or rate: It has dimensions of money per time and typical units of dollars per year. (A dimension is general and independent of the system of measurement, whereas the unit is how that dimension is measured in a particular system.) Comparing net worth to GDP compares a monetary amount to a monetary flow. Because their dimensions differ, the comparison is a category mistake [] and is therefore guaranteed to generate nonsense.

Problem 5.1 Units or dimensions?
Are meters, kilograms, and seconds units or dimensions? What about energy, charge, power, and force?

A similarly flawed comparison is length per time (speed) versus length: "I walk $1.5\,\mathrm{m\,s^{-1}}$—much smaller than the Empire State building in New York, which is 300 m high." It is nonsense. To produce the opposite but still nonsense conclusion, measure time in hours: "I walk 5400 m/hr—much larger than the Empire State building, which is 300 m high."

I often see comparisons of corporate and national power similar to our Nigeria–Exxon example. I once wrote to one author explaining that I sympathized with his conclusion but that his argument contained a fatal dimensional mistake. He replied that I had made an interesting point but that the numerical comparison showing the country's weakness was stronger as he had written it, so he was leaving it unchanged!

A dimensionally valid comparison would compare like with like: either Nigeria's GDP with Exxon's revenues, or Exxon's net worth with Nigeria's net worth. Because net worths of countries are not often tabulated, whereas

Comments on page 2

I really like this example! It's a nice break from all the engineering stuff which is nice but can get repetitive...

Bam! This was pretty excellent. Simple and devastating, lol.

Love this!

I like that both extremes are exemplified.

This note would make more sense one sentence earlier when this distinction is first made.

What are the little brackets for?

Probably a missing citation?

Is this a frequently used term I should know? In context, it sounds like it just means comparing two incomparable things.

Chapter 5: Dimensions or How to Not Generate Nonsense

You should know something about this Paul! :P

I generally think of dimensions as the most simple terms a measurement can be put in-so force has dimensions of "Mass * length/time^2), rather than calling it a dimension in and of itself.

This is an interesting point – are compound dimensions also dimensions?

This seems ludicrous, but I could have easily missed the GDP / net worth comparison. I like the absurd example to point out just how terrible the comparisons are.

However, it might be worthwhile to note something like "it takes me 200 seconds to walk one Empire State Building's worth of distance" or "Nigeria generates nearly the same amount of value in one year as Exxon is worth at any given moment."

Yup. I think the part that seemed way off in this one to me was the use of "speed smaller than length" - lol. The GDP example probably looks as crazy at first glance to an economist that has to think in net worth / GDP every day.

Chris, your summary of the first example makes a lot of sense.

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I think this gets to the point that most journalists are more interested in making emotionally charged statements than mathematically accurate ones

To use a ludicrous example, it would be like a journalist saying that although they cited the wrong source, their information was more powerful coming from a false source so they were leaving it.

To clarify here, "the way that he had worded the (incorrect) argument" and not "because he written it". I wrongly parsed this and had to re-read.

I parsed this correctly, but totally see the room for error. Maybe one more shot at writing this sentence might be a good idea, with a different word or two

I didn't understand the sentence until reading the comments.

corporate revenues are widely available, try comparing Exxon's annual revenues with Nigeria's GDP. By 2006, Exxon had become Exxon Mobil with annual revenues of roughly $350 billion—almost twice Nigeria's 2006 GDP of $200 billion. This valid comparison is stronger than the flawed one, so retaining the flawed comparison was not even expedient!

That compared quantities must have identical dimensions is a necessary condition for making valid comparisons, but it is not sufficient. A costly illustration is the 1999 Mars Climate Orbiter (MCO), which crashed into the surface of Mars rather than slipping into orbit around it. The cause, according to the Mishap Investigation Board (MIB), was a mismatch between English and metric units [15, p. 6]:

> The MCO MIB has determined that the root cause for the loss of the MCO spacecraft was the failure to use metric units in the coding of a ground software file, Small Forces, used in trajectory models. Specifically, thruster performance data in English units instead of metric units was used in the software application code titled SM_FORCES (small forces). A file called Angular Momentum Desaturation (AMD) contained the output data from the SM_FORCES software. The data in the AMD file was required to be in metric units per existing software interface documentation, and the trajectory modelers assumed the data was provided in metric units per the requirements.

Make sure to mind your dimensions and units.

Problem 5.2 Finding bad comparisons
Look for everyday comparisons—for example, on the news, in the newspaper, or on the Internet—that are dimensionally faulty.

5.2 Dimensionless groups

Dimensionless ratios are useful. For example, in the oil example, the ratio of the two quantities has dimensions; in that case, the dimensions of the ratio are time (or one over time). If the authors of the article had used a dimensionless ratio, they might have made a valid comparison.

This section explains why dimensionless ratios are the only quantities that you need to think about; in other words, that there is no need to think about quantities with dimensions.

To see why, take a concrete example: computing the energy E to produce lift as a function of distance traveled s, plane speed v, air density ρ, wingspan L,

Comments on page 3

I feel like I want a stronger statement that we are changing topics here, from matching dimensions to matching units

same here. It would be clearer to include the ratio or the quantities here "GDP / net worth..."

> oops, this was supposed to be lower, down with Ashley's comment on the "dimensionless ratios are useful" paragraph.

I feel like most of the previous section was about dimensions, not units.

have these grey boxes with example problems been here the whole time? I dont remember them from previous readings, and I kind of like them

> They were here at the beginning, but disappeared for about the last 8 readings.

Has anyone found any? It would be fun if people posted examples.

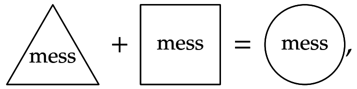
> This should be required homework for tomorrow.

This paragraph confuses me. The 'oil example' is the one with Exxon and GDP, right? The sentence with 'the ratio of the two quantities has dimensions' took me a really long time to get, especially since I was also trying to think if there had been a different 'oil example' at some point.

> Agreed. "The Exxon-Nigeria GDP example" or something similar is perhaps clearer.
>
> > This seems like a unnecessarily round-about way of saying the dimensions of both quantities need to be the same (which will make their ratio dimensionless...)
> >
> > > I agree as well. I have trouble visualizing dimensionless ratios though I know that they are super useful

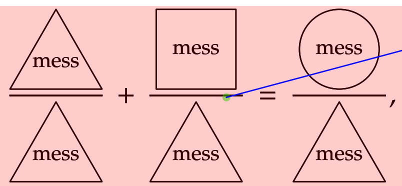
See, I think of it as "If only they had compared things with the same dimensions" - not they should have used a ratio. I think comparison is a more valuable tool than ratios sometimes, because it conveys the actual scale as well as the relative relationship. Admittedly it places a little emphasis on the scale over the ratio because you have to compute it, but I don't see these situations as ratios - I see them as comparisons.

plane mass m, and strength of gravity g. Any meanginful statement about these variables looks like



where the various messes mean 'a horrible combination of E, s, v, ρ, L, and m.

As horrible as that statement is, it permits the following rewriting: Divide each term by the first one (the triangle). Then



The first ratio is 1, which has no dimensions. Without knowing the individual messes, we don't know the second ratio; but it has no dimensions because it is being added to the first ratio. Similarly, the third ratio, which is on the right side, also has no dimensions.

So the rewritten expression is dimensionless. Nothing in the rewriting depended on the particular form of the statement, except that each term has the same dimensions.

Therefore, *any meaningful statement can be rewritten in dimensionless form.*

Dimensionless forms are made from dimensionless ratios, so all you need are dimensionless ratios, and you can do all your thinking with them. As a negative example, revisit the comparison between Exxon's net worth and Nigeria's GDP (Section 5.1). The dimensions of net worth are simply money. The dimensions of GDP are money per time. These two quantities cannot form a dimensionless group! With just these two quantities, no meanginful statements are possible.

Here is a further example to show how this change simplifies your thinking. This example uses familiar physics so that you can concentrate on the new idea of dimensionless ratios.

***meaningful**

"Strength" seems like a strange word to use here.

end quote

Thats a great visualization. Are there more like this?

In my mind, even though the triangle has no dimensions, doesn't the square and circle have an even bigger mess of dimensions? Do we assume those dimensions become meaningless?

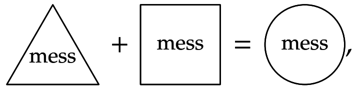
No, because if you are adding two terms, they must have the same dimensions, and the result must have the same dimensions as well. It wouldn't make sense to add: 1m + 2m/s = 3m/s^2. Even though the numbers are correct, the units only make sense if they are all the same.

so the point here is that in order to add things, each term must have the same dimensions. That is kind of an interesting conclusion. This whole 'dividing by messes' business seems like an unnecessarily convoluted way to imply that conclusion... is there another point here?

Isn't the point of dividing by messes not to show that each term must have the same dimensions but rather that we can write the terms in a dimensionless form? Dividing a triangle by a triangle will cancel out the dimensions, which is why you get something dimensionless. That's why you need the dividing part.

But then how is it legal to add them before we divide by triangle?

plane mass m, and strength of gravity g. Any meanginful statement about these variables looks like



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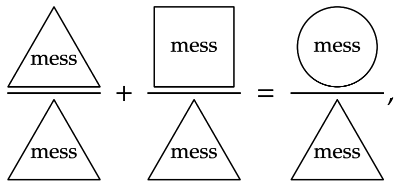

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this section was mind-bending for me, and I'm still not sure I got it. It might help to clarify what you mean by 'meaningful' here. It seems to me that you are saying that "triangle+square=circle" is ugly, but it's meaningful because you can divide them all by triangles and magically all of the dimensions disappear. So, the fact that all of the dimensions disappear is what makes it meaningful, and the fact that it is meaningful is what makes all the dimensions disappear? Sounds circular.

I agree. This was also a bit confusing for me until I read over it a few times.

And if triangle+square=circle is ugly, (triangle/triangle)+(square/triangle)=(circle/triangle) is...something else altogether.

This would be a helpful thing to lecture in class. It's the kind of thing that's hard to put into a single stream of text in a document, but might be very easy to hear while looking at a blackboard as its being marked up.

This sounds dangerous. For example GDP Nigeria/Exxon yearly revenue is a dimensionless constant-and so is Net Worth Nigeria/Net Worth Exxon-but they are not the same. Moving too quickly to dimensionless constants could lead you to forget where your numbers came from, and what they correspond to in the real world.

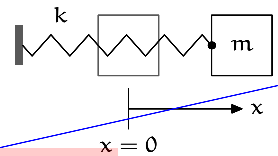
One compares relative worths, and one compares relative yearly revenues. So long as you don't say they compare monetary significance - you are safe. It's important that an accurate descriptor of the quantities behind a ratio is still available.

a positive example might be more useful here. I hope there's one coming up!

***meaningful**

Great way to check yourself. You should emphasize this more maybe?

The problem is to find the period of an oscillating spring–mass system given an initial displacement x_0, then allowed to oscillate freely. The relevant variables that determine the period T are mass m, spring constant k, and amplitude x_0. Those three variables completely describe the system, so any true statement about period needs only those variables.



Since any true statement can be written in dimensionless form, the next step is to find all dimensionless forms that can be constructed from T, m, k, and x_0. A table of dimensions is helpful. The only tricky entry is the dimensions of a spring constant. Since the force from the spring is $F = kx$, where x is the displacement, the dimensions of a spring constant are the dimensions of force divided by the dimensions of x. It is convenient to have a notation for the concept of 'the dimensions of'. In that notation,

Var	Dim	What
T	T	period
m	M	mass
k	MT^{-2}	spring constant
x_0	L	amplitude

$$[k] = \frac{[F]}{[x]},$$

where [quantity] means the dimensions of the quantity. Since $[F] = \mathrm{MLT}^{-2}$ and $[x] = \mathrm{L}$,

$$[k] = \mathrm{MT}^{-2},$$

which is the entry in the table.

These quantities combine into many – infinitely many – dimensionless combinations or groups:

$$\frac{kT^2}{m}, \frac{m}{kT^2}, \left(\frac{kT^2}{m}\right)^{25}, \pi\frac{m}{kT^2}, \ldots.$$

The groups are redundant. You can construct them from only one group. In fancy terms, all the dimensionless groups are formed from one *independent* dimensionless group. What combination to use for that one group is up to you, but you need only one group. I like kT^2/m.

So any true statement about the period can be written just using kT^2/m. That requirement limits the possible statements to

How, in general, do we know that we have all of the variables that describe a particular system? Isn't it possible that there are some we haven't thought of?

I think the point is if we haven't thought of it is irrelevant. We'll make most of them dimensionless anyway.

That makes no sense to me. It's like saying that anything not immediately obvious when we think of characterizing a system is, then, inherently unimportant because we couldn't think of it.

Actually, this is a question I often have as well. In thinking about any system, when can I be sure I've accounted for all of the relevant variables?

And when can we make decisions about how far down to abstract? Working with a mass-spring system, RLC circuit, or drag we have well defined equations and physical understanding. But let's say we were doing an estimation problem with a baseball player batting the ball. Spin, wind, other weather, and countless other "weird" / "non-obvious" / "we could see them totally not being significant" variables of the system still exist - how do we know which ones to think about in comparing or estimating?

hmmm, yeah, here I think Sanjoy is /defining/ his system as one where these are all of the variables, and he's saying that he can hold everything else constant. Figuring out what the important variables are is not covered here, and it would be cool if it were.

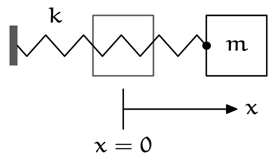
seem to be switching out "true" for "meaningful" here. this is interesting to ponder in several respects...

I think this goes back to the fact that for any combination of terms in a sum, all of the terms and the final result must have the same dimensions. Otherwise it cannot be "true," because the dimensions wouldn't match. So in any "true" statement, it must be possible to divide out that dimension from both sides of the equation to get a dimensionless form.

I think the order is still confusing. The first use of it implied accuracy and thoroughness, rather than factuality. It still feels like we're stretching it a bit here.

In publication it would be helpful to bold this, to distinguish it from the three variables that are relevant to it.

is there a standard set of dimensions we can use? I would have found it natural to use F/L here for the spring constant, but would have found later that that was not as convenient as the version you chose.



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Var	Dim	What
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So any true statement about the period can be written just using kT^2/m. That requirement limits the possible statements to

A visualization like the one Juliana liked would be really nice to have here.

It would be worth noting that x0 doesn't show up in any of these dimensionless combinations, and explaining why.

Is something missing between these two commmas?

I thought that according to Buckingham-Pi, you could have more than one depending on the number of variables you start with.

How and why did we pick this? What makes this a possible group? Are there things that would have been wrong, had we chosen them?

I don't think so, this is just doing some simplification now that we could do later. For example we could have used 3(kT^2/m)^25. This would then lead to 3(kT^2/m)^25 = C, which simplifies to kT^2/m = (C/3)^(1/25) = C': just a different constant.

You need some combination of the terms T, m, k, and x0 that, when multiplied together, have no dimension. Substituting in the dimensions for each term in kT^2/m: the combination has dimensions: [MT^-2][T^2]/[M], which simplifies to being dimensionless.

$$\frac{kT^2}{m} = C,$$

where C is a dimensionless constant. This form has two important consequences:

1. The amplitude x_0 does not affect the period. This independence is also known as simple harmonic motion.
2. The constant C is independent of k and m. So I can measure it for one spring–mass system and know it for all spring–mass systems, no matter the mass or spring constant. The constant is a universal constant.

The requirement that dimensions be valid has simplified the analysis of the spring–mass system. Without using dimensions, the problem would be to find (or measure) the three-variable function f that connects m, k, and x_0 to the period:

$$T = f(m, k, x_0).$$

Whereas using dimensions reveals that the problem is simpler: to find the function h such that

$$\frac{kT^2}{m} = h().$$

Here $h()$ means a function of no variables. Why no variables? Because the right side contains all the other quantities on which kT^2/m could depend. However, dimensional analysis says that the variables appear only through the combination kT^2/m, which is already on the left side. So no variables remain to be put on the right side; hence h is a function of zero variables. The only function of zero variables is a constant, so $kT^2/m = C$.

This pattern illustrates a famous quote from the statistician and physicist Harold Jeffreys [17, p. 82]:

> A good table of functions of one variable may require a page; that of a function of two variables a volume; that of a function of three variables a bookcase; and that of a function of four variables a library.

Use dimensions; avoid tables as big as a library!

Dimensionless groups are a kind of invariant: They are unchanged even when the system of units is changed. Like any invariant, a dimensionless group is an abstraction (Chapter 2). So, looking for dimensionless groups is recipe for developing new abstractions.

my thought process here: why is it not? because it's not in the ratio. why is it not in the ratio? then I had to go up a few paragraphs and infer it. It would be worth mentioning above that x0 doesn't show up in any of the dimensionless combinations above.

Wait.. how does this follow?

I would like some clarification here, too.

I agree this is confusing. I think he means that C is the only thing on the right side of the equation. Thus, no matter which spring-mass system we are looking at, kT^2/m will always be C (i.e. not kC or C/m or something like that). The constant is actually a constant, it doesn't depend on which system we're looking at.

To further clarify, I think it ties back to our gravity example from the previous reading. No matter what k and m are, the constant demands to be met and will not be changed. What does change is the period (T^2).

Im still confused, you can rearrange terms so that C is will be C*m. The only way to get around this problem would be for the ratio of k/m to be constant. Is that true?

So after we find C, we take the sqrt of C*m/k to find T? If this is true, my question is how would it be possible to find C now? It seems like we just pushed the problem of messy dimensions back. Now instead of having to measure things like amplitude, we have to measure this arbitrary constant.

A constant, right?

Nevermind, clarified at the end of the paragraph.

This class is constantly making me rethink what how I perceive the world around me.

We used a good example of this in transport: that running an experiment at varying only one constraint would take, say, 10 graphs to show how that constraint affects the outcome. Varying two constraints would take 10^2, and three would take 10^3...but they can all be collapsed into one dimensionless line.

an SD card

Should this say, "Dimensionless quantities"?

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Is there any great reason why we didn't do this section earlier? It seems as though it might have been useful for the past couple of months. I understand that the course is structured such that lossless compression, like this, makes up the middle third, but maybe some early exposure would be beneficial.

As would some examples.