CONTROL OF NONAXISYMMETRIC MAGNETIC FIELD PERTURBATIONS IN TOKAMAKS

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Received June 16, 2010
Accepted for Publication September 13, 2010

The theory of control of nonaxisymmetric perturbations is dominated by the wide sensitivity range of a tokamak plasma to externally produced magnetic perturbations. External perturbations are characterized by their normal magnetic field $\vec{B}_x \cdot \hat{n}$ on the unperturbed plasma surface. The first spatial distribution of $\vec{B}_x \cdot \hat{n}$ on the unperturbed plasma surface in a sensitivity series is that distribution that at the smallest amplitude has a significant effect on plasma properties. The second distribution of $\vec{B}_x \cdot \hat{n}$ in that series is the distribution to which the plasma has greatest sensitivity while being orthogonal to the first. Two distributions are orthogonal if the integral of their product over the unperturbed plasma surface is zero. Only a limited number of distributions in the sensitivity series can be driven to an unacceptable amplitude by credible construction errors in ITER. Essentially any external coil set that produces a nonaxisymmetric magnetic field of adequate strength with a controllable toroidal phase can null the drive for the distribution of highest plasma sensitivity. However, the simultaneous nulling of not only the first but also of a number of other distributions in the sensitivity series is far more difficult. It is the properties of these distributions of secondary importance that determine both the machine tolerances that are required for successful control and the adequacy of a given set of error field control coils. Nonaxisymmetric fields can also have beneficial effects such as the control of edge-localized modes. Implementation requires driving a normal field distribution to which the beneficial effect is sensitive while not driving detrimental distributions of high plasma sensitivity.

KEYWORDS: *magnetic field errors, resistive wall mode control, nonaxisymmetric perturbations to tokamaks*

Note: The figures in this lecture are in color only in the electronic version.

I. INTRODUCTION

Tokamaks are very sensitive to nonaxisymmetric magnetic fields. Perturbations as small as 10^{-4} of the main magnetic field can cause tokamaks to disrupt, which was interpreted in Ref. 1 (Sec. 2.4.1) as a requirement for tight construction tolerances, $\sim 5 \times 10^{-5}$ in ITER. Tight tolerances have large financial and schedule implications, so the achievement of adequate error field avoidance purely by the accuracy of construction is generally considered unrealistic.

Weak nonaxisymmetric perturbations to the external magnetic field are discussed in three contexts, which are closely coupled by common physics: (a) magnetic field errors,[2,3] which are unintended asymmetries; (b) the resistive wall mode[4–6] (RWM), which is an ideal kink instability of the plasma that grows on the resistive time scale of the wall; and (c) beneficial magnetic perturbations, as for the elimination of edge-localized modes[7] (ELMs).

To achieve its fundamental goals, ITER must deal successfully with magnetic field errors. Resistive wall modes can be avoided in ITER by restricting the operating space, though this would prevent ITER from assessing regimes that are thought to be required for practical tokamak fusion power.[1,8,9] Edge-localized modes must be addressed for ITER to have a reasonable operating life in plasma regimes of primary interest, though alternative strategies for addressing ELMs have been suggested.[10]

Three effects are thought to define the tolerable level of nonaxisymmetric field errors: (a) the drive for magnetic islands at the rational surfaces, which is called the resonant effect[11,12]; (b) the breaking of the axisymmetry of the field strength giving toroidal rotation damping, which is called the nonresonant effect[13]; and (c) the dislocation of the plasma strike points on the divertor to regions not designed to take the implied loads.[14] Different plasma

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① **MATHEMATICS I**
LINEAR ALGEBRA

Doctoral Qualifying Exam
May 2006

1. Consider the Householder reflection matrix

$$H = I - 2\mathbf{u}\mathbf{u}^T$$

where I is the $n \times n$ identity matrix and $\mathbf{u}$ is any unit vector in $\mathbb{R}^n$ (i.e. $\sqrt{\mathbf{u}^T\mathbf{u}} = ||\mathbf{u}||_2 = 1$)

(a) Find H^k where k is a positive integer.

(b) Find the Null space of H for all vectors $\mathbf{u}$.

(c) With very little work, show that H can always be diagonalized.

(d) Find all eigenvalues and eigenvectors of H (the eigenvalues can be found exactly and the eigenvectors can be described in terms of $\mathbf{u}$) (hint: a sketch or example in $\mathbb{R}^2$ of $H\mathbf{v}$ for any vector $\mathbf{v}$ might be useful).

(e) What is $\det(H)$?

(f) Show that $e^H = \cosh(1)I + \sinh(1)H$.

(g) Last one: Find the SVD of H.